ON COMPLEX LOADING AND PROSPECTS OF PHENOMENOLOGICAL APPROACH TO MICROSTRESS RESEARCH

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1. In inelastic rigid bodies the relation between stress and deformation in each moment of the process of deformation inevitably depends on the whole sequence of events in this process, or, as one says, on "the loading path". In view of the variety of rheological properties for rigid bodies, it seems to be unrealistic to originate a common theory, covering all possible cases. Therefore, in this paper the question will be raised only about the most significant version of the theory of complex loading, relating to bodies which possess initial isotropy and in which forces, resisting plastic deformation, do not depend on time.

With certain idealizations (see below) such a theory, which obtained the name theory of flow [plasticity], is suitable for many metals and their alloys at moderate temperatures. The independence of dissipating forces, arising from plastic deformation, on time in essence implies that these forces are of the nature of dry friction.

The fundamental feature of the force of friction as it acts on a moving particle is that it always acts along the tangent to the trajectory of motion in a direction opposite to the velocity. This property of friction determines the tensor structure of relation between stress and deformation in the theory of flow and assumes the form

$$T_{ij} = T\frac{d\varepsilon_{ij}^p}{d\lambda}, \qquad d\lambda = \sqrt{d\varepsilon_{ij}^p\, d\varepsilon_{ij}^p}, \qquad T = \sqrt{T_{ij}T_{ij}} \tag{1.1}$$

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Here ε^p_{ij} is the plastic deformation tensor (which in the following will be known as the "deviator" if we assume that the body is plastically incompressible); T_{ij} is the tensor ("deviator") for dissipation forces of plastic resistance. Equations (1.1) express the condition of tangency to the trajectory of the plastic deformation by the dissipation forces tensor, while resisting plastic deformation.

If we assume that there are no other forces resisting plastic deformation except T_{ij} and that the friction is constant along the trajectory of deformation, i.e. what the invariant $T = T_0$ = const, then T_{ij} is necessarily identified with the stress "deviator" σ'_{ij} and then (1.1) becomes the most elementary theory of flow-theory of Reuss. If we retain the assumption that the resistance of plastic deformation is of purely dissipative character, but consider that the friction T depends on the position of points on the trajectory of deformation, then we are lead to the theory of flow with isotropic hardening [1].

However, it is not at all necessary to suppose that the resistance to plastic deformation has an entirely dissipative character. In the process of plastic deformation internal elastic forces of resistance can arise as a consequence of its irregularity. We denote the tensor of the external effect of this force by s_{ij}. In this case the tensor of forces of dissipation may be expressed by Equation

$$T_{ij} = \sigma_{ij}' - s_{ij} \tag{1.2}$$

If we assume that σ_{ij}' are connected with ε^p_{ij} by a linear relationship, analogous to Hooke's law, and that $T = T_0$, we obtain from (1.1) the elementary varient relating to the translational theory of flow which is sometimes known as the theory with an ideal Bauschinger effect. The last theory may be generalized, if we relinquish the assumption that T is constant along the trajectory of deformation [2 to 4]. As is obvious, an intuitive representation of dissipative forces, resisting plastic deformation in the basis of the theory of flow, is assumed as in the case of forces of dry friction. This finds its mathematical expression in the requirement of tangency of the tensor of dissipation forces T_{ij} to the trajectory of plastic deformation, and also in the requirement of independence of this tensor of the time of deformation.

The indicated versions of theory of flow generate the trend in the theory of plasticity, which should be called the fundamental one, even though, certainly, they do not exhaust all the wealth of ideas suggested by different authors for description of deformation processes of initially isotropic inelastic bodies.

Experimental evidence to test different versions of the theory of flow is very extensive and its data are in general favorable for the theory (see, for example, [5 and 6]). Even its simplest versions, as a rule, coincide satisfactorily with experiments. Translational theories which take into account the Bauschinger effect, give a noticeable accuracy only for essentially complex loading paths (for example, for passing around the region of elastic deformations or changing the signs of the loading). However, in discussing the coincidence of predictions of theory of flow with experiment data, it is necessary to note that the extent of validity of these assertions depends on assumptions with which the plastic deformations are measured (i.e. on their smallest value, beginning with which we agree to observe them).

In this respect the works of Iagn and his followers [7 and 8] are typical, in which it was established that for very small assumptions for plastic deformation (of order 0.001%) not only very simple but even very complex forms among the suggested versions of the theory of flow do not give accurate representation of the features of plastic deformation. Under these assumptions the boundary of elastic deformations has a complicated form, changing essentially in the process of deformation. As the assumption is extended the observed pattern is beginning to become simplified and for assumption of order 0.05% come close to the fundamental predictions of versions of the theory of flow. For subsequent increase of assumption again a deviation of theory from experiment begins to occur. Thus, for example, in the theory of flow (and in the theory of plasticity in general) it is asserted that if, beginning with some active loading, unloading and then once again loading occurs, then the new flow limit will be that maximum stress σ_s, which was reached in the process of loading. In experiments, however, if we admit large assumption (of order 0.2%, which is generally accepted in engineering) we find that the yield stress for repeated loading is somewhat larger than σ_s. Conversely, if we consider small assumption (of order 0.01% or still smaller), then the yield stress for repeated loading is shown to be below σ_s. Therefore, if we speak of a good agreement of theory of flow with experiment it is necessary without fail to add that it occurs only for sufficiently rough processing of experimental data.

Hence, it follows that under the differentials in formulas of flow, as a matter of fact, one must imply finite increments of deformation of the order of such tolerances, from which assumptions lying at the basis of a theory are justified. The last does not interfere with the validity of the theory of flow to predict the general picture of the dependence of the trajectories of deformation on the loading trajectories, but it is not able to include local effects in the immediate proximity of sharp changes of the direction of loading. Insufficient accuracy of the theory of flow in the description of quantities, changing rapidly within relatively small changes of plastic deformation, probably, is the main cause of its inadequacy in connection with problems of stability of equilibrium of elastic plastic bodies. As is well known [9 and 10], for the problem area better coincidence with experiment is given by the theory of small plastic deformations. Why this is so is difficult to say, but one thing is evident – there is no reason to be astonished by the discrepancy of the flow theory and experiments in the area of stability of equilibrium of elastic plastic bodies.

2. Although in the description of the general picture of plastic deformation in the theory of flow already has achieved considerable success, nevertheless its perfection should be continued as with the goal of further approaching theoretical results with experiments in respect to prediction of the deformation curve form representation of the loading curve, as in particular with the purpose of studying microdeformation and microstresses arising in bodies from their elastic-plastic deformation. The appearance of these microdeformations and microstresses depends on microscopic nonhomogeneous elastic and plastic properties of polycrystals, and also on the imperfections in the structure of their crystal lattice points, i.e. dislocations. In the theory of elasticity and theory of plasticity stress and deformation are usually averaged in the limits of elemental volumes containing sufficiently large number of crystal lattice points, and a relation between averaged stresses and averaged deformations are established. These stresses and deformations in the sequel will be called macroscopic.

However, for formulation of a law for this relation we cannot avoid to deal with microscopic nonhomogeneity of stress and deformation field, since the work accomplished by self-equilibrating microstresses in correspondence to their microdeformations is comparable with the work of averaged stresses on averaged deformations. Particularly typical of this are examples of numerous experiments for the measure of heat, evolved during macroscopic homogeneous deformation ([11 and 12] and others). The comparison of the work corresponding to this heat with the work expended on the plastic deformation shows that the mechanical equivalent of heat evolved is always appreciably lower than the expended work (by 5 – 8% , depending on magnitude of deformation). From this it follows that in a homogeneously deformed elastic-plastic body after removal of all its stresses there remains an elastic field of deformations and corresponding to it a field of residual stresses, the appearance of which may be explained only by microscopic inhomogeneity of mechanical properties of the body.

To this one should add that for homogeneous macrostresses and macrodeformations in experiments with specimens beyond the yield stress, there undoubtedly arise not only elastic but also plastic nonhomogeneous microdeformations which in experiments of the type [11 and 12] are not detected, but for which is expended work apparently comparable in magnitude with the work expended for elastic residual microdeformations, so that in realty from all the work expended on plastic deformation of the body, probably not less than 10 – 15% may be due at the expense of selfequilibrating microstresses and their corresponding microdeformations.

3. Work expended in microdeformations is found to be comparable with work expended in averaged deformations. This property allows the introduction of the macroscopic tensor, which is the statistical characteristic of microstresses and permits one to consider the influence of the latter in the relationship between macroscopic stresses and deformations.

We consider a sufficiently small, but finite volume element of a polycrystalline body containing a large number of crystalline lattice points. The work required for a change of deformation of this volume element, relative to the unity of its volume, is expressed by Equation

$$dR = \frac{1}{V}\int \sigma_{ij} d\varepsilon_{ij} dV \tag{3.1}$$

in which the integral is extended to include all volume of the element. We assume stresses and deformations in the form

$$\sigma_{ij} = \sigma_{ij}^{\circ} + \sigma_{ij}^{*}, \qquad \varepsilon_{ij} = \varepsilon_{ij}^{\circ} + \varepsilon_{ij}^{*} \tag{3.2}$$

Here σ_{ij}° and ε_{ij}° are constant tensors in the limits of the volume element under consideration, they are equal in value to the average stresses and deformations

$$\sigma_{ij}^{\circ} = \frac{1}{V}\int \sigma_{ij} dV, \qquad \varepsilon_{ij}^{\circ} = \frac{1}{V}\int \varepsilon_{ij} dV \tag{3.3}$$

As regards σ_{ij}^{*} and ε_{ij}^{*}, their integrals in the limits of the volume element are equal zero

$$\int \sigma_{ij}^{*} dV = \int \varepsilon_{ij}^{*} dV = 0 \tag{3.4}$$

Substituting (3.2) in (3.1) and considering (3.4) we arrive at Equation

$$dR = dR^{\circ} + dR^{*} = \sigma_{ij}^{\circ} d\varepsilon_{ij}^{\circ} + \frac{1}{V}\int \sigma_{ij}^{*} d\varepsilon_{ij}^{*} dV \tag{3.5}$$

In this equation the first term on the right-hand side is the work of the averaged stresses on the increments of averaged deformations, and the second is the work of selfequilibrating stresses on the increments of deformations corresponding to them. For microinhomogeneous elastic and plastic properties of bodies the second integral always differs from zero. We cannot, as was already indicated, neglect it.

For volume elements containing a sufficiently large number of crystalline lattice points; the work of selfequilibrating stresses, relative to unit volume dR^*, should be considered independent on both the shape and the size of the element, i.e. the second term of Equation (3.5), for given $\overset{\circ}{\sigma}_{ij}$, $\overset{\circ}{\varepsilon}_{ij}$, and $d\overset{\circ}{\varepsilon}_{ij}$ and for given sequence of events in loading, is for every body a definite quantity, characterizing its microstructure. Let us introduce a symmetric macroscopic tensor of the second rank, having measurement dimensions of stress, connecting it with dR^* by the equality

$$dR^* = \sigma_{ij}^{**} \, d\overset{\circ}{\varepsilon}_{ij} \tag{3.6}$$

The fundamental property of the given tensor, arising from (3.5) and (3.6), consists in that the specific work done by it on increments of macroscopic deformations is equal to the specific work of all microstresses on the micro-deformations corresponding to them. Formula (3.6), it is understood, does not determine completely σ_{ij}^{**}. However, it shows that such a tensor may be introduced and that it always differs from zero. Below, on the basis of a series of physical arguments, are indicated means to its concrete definition.

Substituting (3.6) into (3.5) we obtain Equation

$$dR = (\overset{\circ}{\sigma}_{ij} + \sigma_{ij}^{**}) \, d\overset{\circ}{\varepsilon}_{ij} = S_{ij} d\overset{\circ}{\varepsilon}_{ij} \tag{3.7}$$

Thus, in the expression for increment of specific work of deformation of polycrystalline material, along with averaged stresses σ_{ij}°, there enters also the macroscopic tensor σ_{ij}^{**} which accounts for microstresses.

4. Let us divide now, as is usual, the macroscopic deformation into its elastic and plastic parts $(\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p})$.

Then

$$dR = S_{ij} \, (d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p}) \tag{4.1}$$

In accordance with this equation, the stresses S_{ij}, applied to volume element of the body from outside, overcome the internal forces resisting elastic deformation, as well as those resisting plastic deformation. Since S_{ij} do the work in increments of elastic deformations it follows (if it is assumed that elastic deformations do not depend on the path of loading) that

$$S_{ij} = \frac{\partial \Phi}{\partial \varepsilon_{ij}^{e}} \tag{4.2}$$

Here Φ is a scalar function of the invariants of tensor ε_{ij}^{e} and is a stress potential.

As regards the internal forces resisting plastic deformation, they may be divided into the following three categories.

a) Dissipative forces $(-T_{ij})$ averaged with respect to elementary volume V, having the character of dry friction, and thus connected with macroscopic plastic deformations ε_{ij}^p by Equation (1.1).

b) Elastic microstresses, caused by plastic deformation, determined by it and vanishing with it. The macroscopic tensor corresponding to them shall be designated by s_{ij}.

c) Microscopic forces of the dry friction type. The tensor of these stresses at each point inside the isolated volume V is tangent to the local trajectory of plastic deformation (to the microtrajectory) from which, however, it does not follow that the corresponding macroscopic tensor $(-p_{ij})$ is directed along the tangent to the trajectory of macroscopic plastic deformations ε_{ij}^p. It may have, generally speaking, also another direction.

On the basis of what was said above

$$S_{ij} = \overset{\circ}{\sigma}_{ij} + \sigma_{ij}^{**} = T_{ij} + s_{ij} + p_{ij} \tag{4.3}$$

Hence

$$T_{ij} = \overset{\circ}{\sigma}_{ij} = S_{ij} - s_{ij} - p_{ij} \tag{4.4}$$

The work of stresses s_{ij} on a closed cycle of plastic deformations, as was indicated above, is equal to zero. In view of this

$$s_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}^p} \tag{4.5}$$

where Ψ is a function of plastic deformations ε_{ij}^p.

The tensor s_{ij} was introduced earlier: it is present in all versions of theory of flow, which take into account anisotropic hardening. It was called tensor of microstresses for the first time in paper [4].

The reasoning presented above clarifies the meaning of this macroscopic tensor and the justification for its designation. However, from this reasoning it is also implied that besides the tensor s_{ij}, representing microstresses, which do work on elastic microdeformations, on the same footing also the tensor p_{ij} should appear, which is the representative of microstresses doing work in plastic microdeformations. Until now the term S_{ij}, apparently, was not taken into account, although in all probability contributions to specific work of deformation of s_{ij} and p_{ij} are approximately equal. If p_{ij} is not taken into account, then we are lead to the theory of flow with moving and, possibly, expanding boundary of the region of elastic deformations. The coordinates of the center of this region will be the components of the tensor s_{ij}. The deficiency of this theory, which was called to author's attention by A.A. Vakulenko, is that it leads to the conclusion that as a result of cyclic deformation, closed with respect to stresses and also with resect to deformations, the material again becomes initially isotropic, which is, generally speaking, not in agreement with experiments.

The experiments indicate that after a deformation, closed with respect to both stresses and deformations in the body, as a rule, deformational anisotropy is retained. Thereby the center of the region of elastic deformations is displaced along some tensor trajectory, not reaching its initial position, which is an argument in favor of the necessity of retaining in Equation (4.4), in addition to s_{ij}, also the second term p_{ij}, which, being representative of microstresses doing work in plastic microdeformations, does not vanish in a closed cycle of macroscopic deformation.

Since the coordinates of the center of the region of elastic deformations are defined by the tensor $\sigma_{ij}^{**} = s_{ij} + p_{ij}$, then the retaining of p_{ij} in Equation (4.4) guarantees the possibility of describing the deformational anisotropy, remaining in the body under cyclic deformations, closed with respect to both stresses and deformations.

The tensor s_{ij}, as was already mentioned, is related to the macroscopic plastic deformations ε_{ij}^{p} by the finite relations (4.5). As regards p_{ij}, then even though it should be related to plastic deformations, this relation, however, must have the form of nonintegrable differential relationships (in view of the dissipative character of forces, corresponding to p_{ij}).

The direction, along which the form of the indicated relations must be sought is intimated by the already mentioned experimental fact, according to which the center of the region of elastic deformations following the plastic deformations is somewhat lagging in their evolution. But the same characteristics, as is well known, are exhibited by the tensor of plastic deformations with respect to the stress tensor.

Hence there are reasons to look for relationship between p_{ij} and the plastic deformations ε_{ij}^{p} in the form

$$dp_{ij} = \varepsilon_{ij}^{p}\, d\chi \tag{4.6}$$

Here χ is a function of T. The latter follows from the fact that in the absence of increment of plastic deformations the tensor p_{ij} must remain constant.

The author does not assert that the form of the relationship between p_{ij} and ε_{ij}^{p} (4.6) is optimum from the point of view of the possibility of decreasing the discrepancy of theory and experiments, but this, apparently, is the simplest among possible conjectures. In the final analysis it is precisely the experiment which must prompt the most rational choice of a relation between p_{ij} and ε_{ij}^{p}.

5. The indicated avenues of refining the theory of flow, being strengthened by systematic experiments,allow its perfection in the direction of improved description of anisotropic hardening, the external appearance of which is the Bauschinger effect.

This will give one the possibility to successfully apply the theory to more complicated loading and deformation paths, than it is possible to do now. But the most important possiblity which can be expected from the indicated development of the theory of flow is the possibility of investigating certain properties of microstresses, arising from the plastic deformation of polycrystals. In metallurgical science the role of microstresses has been considered significant already for a long time. They are looked upon as the cause which induces microcracks in the material and their subsequent development.

At present that physical theory of internal microstresses is being successfully developed, which is based on contemporary models of the real structure of solid bodies and which rests on the apparatus of dislocation theory. Attempts are being made to study microstresses and microdeformations by different contemporary methods of experimental physics, in particular, by methods using X-rays.

Without in the least rejecting all these directions in a study of microstresses and microdeformations, we call attention here to still another (and most simple) possibility, namely, the possibility of studying microstresses within the framework of the phenomenological theory of plasticity.

It is found that microstresses send their representative into the world of macroscopic phenomena, observed in strength of materials laboratories on well-known and wide-spread testing machines. Such a representative is the macroscopic tensor σ_{ij}^{**}, whose work in averaged deformations is equal to the work of microstresses in microdeformations. The indicated fundamental property of σ_{ij}^{**}, as well as some assumptions based on experimental data, permit its fairly rigorous determination by means of its influence on the pattern of macroscopic deformation. The use of this tensor, which should have been called "tensor of external appearance of microstresses", if this name would not be too long, permits one to constitute an averaged (in some sense) representation of the pattern of microstresses and of its dependence on the loading path.

One may object that such averaged representation is insufficient, since it does not in any way estimate the maximum value of microstresses. Besides, in view of considerable nonuniformity o. the field of microstresses, their deviations of maximum value from averaged values may be very large. This observation is completely true. The phenomenological approach does not give and, obviously, cannot in principle give estimates of the limits of fluctuation of microstresses, which is a serious deficiency. However, even the averaged characteristics of microstresses represent an unquestionable interest. In particular they allow one to obtain an estimate of the work, performed by microstresses as a function of the loading path and of latent elastic energy, accumulated in the body. If the origin of microcracks is caused, apparently, by maximum value of microstresses, then the subsequent development of an already formed crack may, possibly, depend mainly on the averaged elastic energy of microstresses.

But even if one adopts the most cautious and most disadvantageous assumption for the phenomenological description of microstresses, namely that the averaged pattern does not give immediate possibility to estimate the criteria of strength of materials for complex loading, even then the given direction must be followed as an auxiliary method of study of microstresses, supplementing other (physical) methods of their study. Indeed, when finally a sufficiently worked out physical theory will be developed, allowing to consider not only averaged characteristics of microstresses, but also bounds of their fluctuations, then on the basis of this theory one should be able to deduce, in particular, equations tking into account the influence of microstresses on the pattern of microscopic deformations. The given formulas, in their structure will, undoubtedly, be similar to those, which were given above, and the degree of their coincidence with experiments will allow to judge the degree of accuracy of the developed theory.

Therefore one should not discard the possibilities of the phenomenological approach, allowing to obtain some (even if incomplete) information concerning the world of microstresses, which is still hidden from us, on the basis of conventional mechanical testing of specimens, i.e. by means of an experiment which is simple and well developed.

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